METHODS OF SOLVING NONSTATIONARY PROBLEMS IN THE THEORY OF RADIATIVE HEAT EXCHANGE

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The generalized zonal method of Surinov and the method of successive approximations are applied to the study of nonstationary radiative heat exchange.

In this paper, we study nonstationary problems in the theory of radiative heat exchange in emitting systems of arbitrary configuration and sizes bounded by diffusion-emitting and reflecting surfaces and occupied by an absorbing, isotropically scattering medium at rest [1, 2].

The generalized statement of the problem, ignoring heat conduction in the system, is the determination of the temperature of the medium $T(M, \tau)$ as a function of point $M \in V$ and time $\tau \in [0, \tau_1]$ from the solution of the Cauchy problem for the nonlinear integrodifferential equation of the temperature field in the medium [2-5]:

$$\rho(M) c(M) \frac{\partial T(M, \tau)}{\partial \tau} = \alpha(M) \Big[\int_F E_\alpha(N, \tau) \Omega^{(1)}(M, N) dF_N + 4\sigma_0 \int_V \alpha(P) T^4(P, \tau) \chi^{(1)}(M, P) dV_P - 4\sigma_0 T^4(M, \tau) \Big] + q(M, \tau), \quad M \in V; \ \tau \in [0, \tau_1] \tag{1}$$

subject to the initial condition

$$T(M, 0) = T_0(M) \tag{2}$$

Then the other (not given) radiation characteristics are calculated from their integral representations in terms of the radiation resolvents [1, 2].

The function $E_\alpha(N, \tau)$ in (1) is the generalized boundary radiation characteristic [2, 3].

An exact solution of (1) and (2) has not been worked out. In [4] the zonal method of solving nonstationary problems of this type was first proposed, based on an average of (1) over the zone volume and the solution of the Cauchy problem for a system of ordinary differential equations. This method was applied in [6] to the numerical study of the nonstationary radiation field in a cylindrical chamber of finite length occupied by an absorbing, scattering medium.

In the present paper we give a new approach to the zonal solution of nonstationary problems in radiative heat exchange which applies both to the generalized zonal method [5] and the method of successive approximations. The essence of the approach is that unlike the zonal method for nonstationary radiative heat exchange [3], which is based on the direct use of the integrodifferential equation (1), we use an iterative process, outlined as follows.

1. As a zeroth-order approximation, we take the temperature $T_0(M, \tau)$ of the medium at the initial instant of time $\tau = 0$. This approximation determines the temperature of the medium only as a function of the coordinates

$$T_0(M, \tau) = T_0(M). \tag{3}$$

2. If at the initial time $\tau = 0$ the temperature of the medium is given as a continuous function of point M ($M \in V$) then, dividing up V into a finite number m of zones ($V = \sum_{j=1}^{m} V_j$) and carrying out a zonal average of the fourth power of the absolute temperature for $\tau = 0$ according to the expression

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$$T_j^4(\tau) = \frac{1}{V_j}\int_{V_j} T^4(M_j, \tau)\, dV_{M_j} \quad (j = 1, 2, \ldots, m), \tag{4}$$

we can determine (using any of three possible forms of the generalized zonal method [3, 5]) the distribution $\eta_{res.0}(M, \tau)$.

The first step in the iterative process is the substitution in the equation

$$c(M)\rho(M)\frac{\partial T(M, \tau)}{\partial \tau} = \eta_{res}(M, \tau) + q(M, \tau) \tag{5}$$

of the value $\eta_{res.0}(M, \tau)$ obtained above. This equation is solved numerically for initial condition (2), and the first approximation for the temperature of the medium $T_1(M, \tau)$ is found as a function of the coordinates and time.

3. The second step in the iterative process is an average of $T_1^4(M, \tau)$ (fourth power of the absolute temperature of the medium) with respect to zones V_j ($j = 1, 2, \ldots, m$), the calculation of $\eta_{res.1}(M, \tau)$ with the help of the generalized zonal method and the solution of differential equation (5) for initial conditions (2).

4. Using the temperature distribution $T_k(M, \tau)$ obtained in the k-th step of the iterative process we calculate, using any of three forms of the generalized zonal method [3, 5], the volume resultant radiation density $\eta_{res.k}(M, \tau)$ as a function of point $M \in V$ and time τ. In particular, the first two forms of the generalized zonal method give

$$\eta_{res.k}(M, \tau) = \alpha(M)\Big[\sum_{i=1}^{n} E_{\alpha,i}(\tau)\,\Pi^{(1)}(M, F_i) + \sigma_0 \sum_{j=1}^{m} T_{k,j}^4(\tau)\,\mathfrak{A}^{(1)}(M, V_j) - 4\sigma_0 T_k^4(M, \tau)\Big], \tag{6}$$

where n and m are the number of boundaries F_i and volumes V_j of optically and energetically uniform zones into which we have divided up the radiating system. $E_{\alpha,i}(\tau)$ is the value of the generalized boundary radiation characteristic $E_\alpha(N, \tau)$ averaged over zones F_i ($i = 1, 2, \ldots, n$). $T_{k,j}^4(\tau)$ is the value of $T_k^4(M, \tau)$ averaged over the zones V_j ($j = 1, 2, \ldots, m$) according to (4).

Substituting the approximate value $\eta_{res.k}(M, \tau)$ into (5), we find numerically the (k + 1)-approximation $T_{k+1}(M, \tau)$ for the temperature field of the medium as the solution of the Cauchy problem (5), (2). The iterative process is continued until two successive approximations T_k and T_{k+1} do not differ by more than a sufficiently small quantity ε:

$$|T_k - T_{k+1}| < \varepsilon. \tag{7}$$

If we assume that the functions $E_{\alpha,i}(\tau)$ ($i = 1, 2, \ldots, n$) are continuous on the interval $[0, \tau_1]$ and that functions $c(M)$, $\rho(M)$, $\alpha(M)$, $T_0(M)$ are continuous and bounded in V and $q(M, \tau)$ is continuous and bounded in $V \times [0, \tau_1]$, it can be shown that the sequence of approximations $T_k(M, \tau)$ uniformly converges in a sufficiently small time interval $[0, \tau_2] \subset [0, \tau_1]$ to a unique solution $T(M, \tau)$ of the Cauchy problem [3, 6]:

$$c(M)\rho(M)\frac{\partial T(M, \tau)}{\partial \tau} = \alpha(M)\Big[\sum_{i=1}^{n} E_{\alpha,i}(\tau)\,\Pi^{(1)}(M, F_i) + \sigma_0 \sum_{j=1}^{m} T_j^4(\tau)\,\mathfrak{A}^{(1)}(M, V_j) - 4\sigma_0 T^4(M, \tau)\Big] + q(M, \tau); \tag{8}$$

$$T(M, 0) = T_0(M). \tag{9}$$

To prove this convergence, (8) with initial condition (9) is replaced by the equivalent Volterra integral equation

$$T(M, \tau) = T_0(M) + \frac{1}{c(M)\rho(M)}\int_0^{\tau} [\eta_{res}(M, \xi) + q(M, \xi)]\, d\xi \tag{10}$$

and the principle of compact mapping is used [7]. All remaining unknown radiation characteristics are calculated using the generalized zonal method for a given temperature field $T(M, \tau)$ [3, 5].

Below we apply this method to the numerical solution of the nonstationary spatial radiative heat exchange in a cylindrical chamber of finite length, occupied by a uniform absorbing, isotropically scattering medium with attenuation factor $k = \alpha + \beta$.

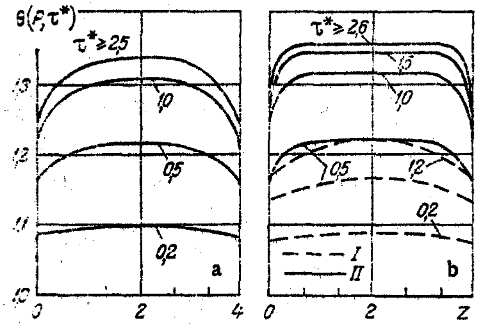

Fig. 1. Nonstationary temperature distribution θ(P, τ*) on the symmetry axis of the system for a purely absorbing medium (p = 0): a) Bu = 1.0, b) Bu = 0.1 (curve I), Bu = 10 (curve II).

We consider the following formulation of the problem: at the initial time $\tau = 0$ the radiating system is taken away from the state of thermodynamic equilibrium by an instantaneous change (jump) in the output of the internal heat sources q from value q = 0 to value $q = (1/2)\eta_c(M, 0) = 2\alpha\sigma_0 T_0^4$. The ends F_1 and F_2 of the cylinder are assumed to behave like perfect blackbodies and be isothermal at temperatures T_1 and T_2 initially equal to the temperature of the medium T_0, while the curved surface F_3 of cylinder is taken to be adiabatic: $E_{res}(N_3, \tau) \equiv 0$.

It is required to determine the nonstationary temperature distribution $T(M, \tau)$ and the volume resultant radiation density $\eta_{res}(M, \tau)$ with respect to the volume V occupied by the radiating system. It is also required to determine the nonstationary surface density distribution of resultant radiation on the ends of the cylinder (F_1, F_2) and the temperature $T(N_3, \tau)$ and the temperature jumps $\Delta T(N_3, \tau)$ on the curved surface F_3 of the cylinder.

From (2), (5), (6) for the temperature field of the medium and the volume resultant radiation density, we have the following expressions suitable for calculation, and in dimensionless form:

$$\frac{\partial \theta_{k+1}(M, \tau^*)}{\partial \tau^*} = q_{res.k}(M, \tau^*) + 0{,}5 \quad (M \in V;\ \tau^* \in [\tau^*_{l-1}, \tau^*_l]); \tag{11}$$

$$\theta_{k+1}(M, \tau^*_{l-1}) = \theta(M, \tau^*_{l-1}) \quad (l = 1, 2, \ldots);\ \theta(M, 0) = 1; \tag{12}$$

$$q_{res.k}(M, \tau^*) = \frac{1}{4}\left\{4 + [\theta_k^4(\tau^*) - 1]\,\tilde{\mathfrak{A}}^{(1)}(M, V)\right\} - \theta_k^4(M, \tau^*), \tag{13}$$

where

$$\tau^* = \frac{4\alpha\sigma_0 T_0^3}{c\rho}\tau;\quad \theta_k(M, \tau^*) = \frac{T_k(M, \tau^*)}{T_0};$$
$$q_{res.k}(M, \tau^*) = \frac{\eta_{res.k}(M, \tau^*)}{4\alpha\sigma_0 T_0^4}. \tag{14}$$

Result (13) for $q_{res.k}(M, \tau^*)$ is obtained under the assumption that the radiating system consists of three boundary zones F_i (i = 1, 2, 3) and one volume zone V. Dimensionless expressions for the other unknown radiation characteristics are derived with the help of the generalized zonal method [3].

The numerical solution of (11) and (12) is done using a single-step difference method

$$\theta_{k+1}(M, \tau^*_l) = \theta(M, \tau^*_{l-1}) + \left\{\frac{1}{4}[4 + (\theta_k^4(\overline{\tau^*_l}) - 1)\,\tilde{\mathfrak{A}}^{(1)}(M, V)] - \theta_k^4(M, \overline{\tau^*_l}) + 0{,}5\right\}(\tau^*_l - \tau^*_{l-1}), \tag{15}$$

where

$$\theta_k^4(\overline{\tau^*_l}) = \theta^4(\tau^*_{l-1});\quad \theta_k^4(M, \overline{\tau^*_l}) = \theta^4(M, \tau^*_{l-1}),\ \text{if}\quad k = 0,$$

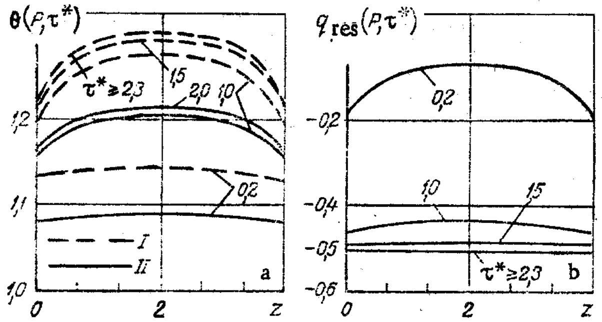

Fig. 2. Nonstationary distributions $\theta(P, \tau^*)$ and $q_{res}(P, \tau^*)$ on the symmetry axis for an absorbing, scattering medium (Bu = 1.0, $p \neq 0$): a) p = 0.4 (curve I), p = 0.8 (curve II); b) p = 0.4.

$$\theta_k^4(\bar{\tau}_l^*) = \frac{1}{2}[\theta_k^4(\tau_{l-1}^*) + \theta_k^4(\tau_l^*)\Big];$$

$$\theta_k^4(M, \bar{\tau}_l^*) = \frac{1}{2}[\theta_k^4(M, \tau_{l-1}^*) + \theta_k^4(M, \tau_l^*)], \quad \text{if } k \geqslant 1.$$

Using (15) the temperature of the medium $\theta(M, \tau^*)$ was determined as a function of point M ($M \in V$) at successive times $\tau_l^* = lh$ ($l = 1, 2, \ldots,$) with step size h = 0.05, and at each time interval $[\tau_{l-1}^*, \tau_l^*]$ no more than two iterations of (15) were required in order to satisfy (7) with $\varepsilon = 0.005$.

In the numerical calculations, the dimensionless height of the cylinder H = H'/R' was taken as fixed and equal to 4.0, while the Buger number Bu = kR' and the parameter $p = \beta/k$ were allowed to vary as follows: Bu = 0.1; 1.0; 10.0; p = 0; 0.4; 0.8. The position of point M in the system was given by two dimensionless coordinates z and ρ, where z = z'/R' is the distance of point M from end F_2 of the cylinder and $\rho = \rho'/R'$ is the distance of point M from the symmetry axis of the system.

The numerical results are shown graphically in Figs. 1-4. In Figs. 1 and 2 the nonstationary temperature distribution $\theta(M, \tau^*)$ and the resultant radiation volume density $q_{res}(M, \tau^*)$ on the symmetry axis of the system are shown, and in Figs. 3 and 4 are shown the nonstationary resultant radiation surface density distribution $\theta_{res}(N_1, \tau^*) = E_{res}(N_1, \tau^*)/(\sigma_0 T_0^4)$ on surface F_1 of the cylinder, as well as the temperature $\theta^4(N_3, \tau^*)$ and temperature jumps $\Delta\theta(N_3, \tau^*)$ on the curved surface F_3.

It follows from these graphs that the radiating system evolves with time toward a stationary state, with the rate of going into the stationary state slowing with an increase in the Buger number and with decrease of the parameter p. It is also seen from Fig. 1 that for large values of τ^*, a nonuniformity arises in the temperature distribution $\theta(M, \tau^*)$ with

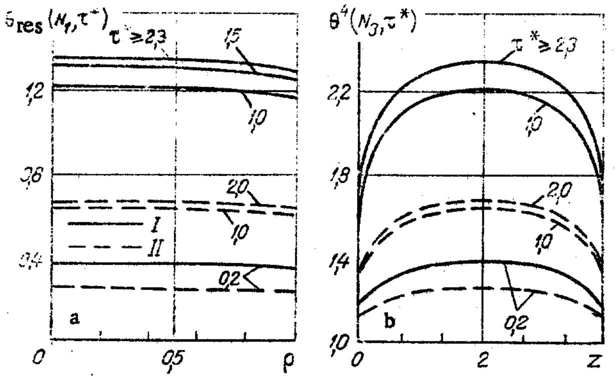

Fig. 3. The dependence of $\theta_{res}(N_1, \tau^*)$ and $\theta^4(N_3, \tau^*)$ on coordinates of points N_1, N_3 and time τ^* (Bu = 1.0): Curve I: p = 0.4; curve II: p = 0.8.

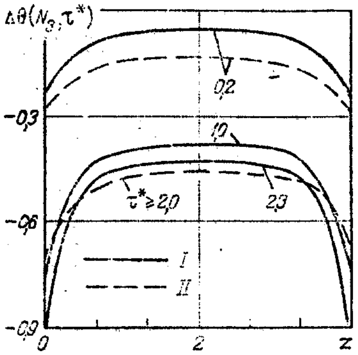

Fig. 4. Nonstationary temperature jump distribution $\Delta\theta(N_3, \tau^*)$ on the curved surface F_3 of the cylinder, Bu = 1.0: Curve I: p = 0.4; curve II: p = 0.8.

respect to height, and this nonuniformity becomes more pronounced near the ends of the cylinder for large values of the Buger number (Bu = 10.0). In the planes z = const, the numerical results show that the temperature distribution $\theta(M, \tau^*)$ and the resultant radiation volume density $q_{res}(M, \tau^*)$ are nearly uniform, with $q_{res}(M, \tau^*) < 0$ and increasing in absolute value with increasing τ^*.

It is seen from Fig. 3a that the resultant radiation surface density $\theta_{res}(N_1, \tau^*)$ is positive, increases with increasing τ^*, and at fixed τ^* is a decreasing function of coordinate ρ of point N_1. We also note that $\theta_{res}(N_1, \tau^*) = \theta_{res}(N_2, \tau^*)$, where $N_1 \in F_1$ and $N_2 \in F_2$ are congruent points.

The numerical results for the temperature jump $\Delta\theta(N_3, \tau^*)$ on the curved surface F_3 of the cylinder show that the stationary value $\Delta\theta(N_3)$ approaches the value 0.5 with increasing p (for a given Bu) and with increasing Buger number for a given p. This result also follows directly from (11) through (13) because $\tilde{\mathfrak{A}}(N_3, V) \to 1$ and $\tilde{\mathfrak{A}}^{(1)}(M, V) \to 4$ when Bu $\to \infty$ and p $\to$ 1.

NOTATION

$T(M, \tau)$, absolute temperature of the medium at point M and time τ; η_{res}, resultant radiation volume density; q, volume density of sources (sinks) of heat; E_α, generalized boundary radiation characteristic; σ_0, Stefan–Boltzmann constant; k, attenuation coefficient of the medium; α, volume absorption coefficient of the medium; β, volume scattering coefficient of the medium; c, specific heat capacity of the medium; ρ, density of the medium; $E_{\alpha,i}$, generalized boundary radiation characteristics averaged over zone F_i; T_j^4, fourth power of the absolute temperature of the medium averaged over the volume zone V_j; θ, dimensionless temperature of the medium; q_{res}, dimensionless resultant radiation volume density; τ^*, dimensionless time; $\Omega^{(1)}$ and $\chi^{(1)}$, resolvents; $\tilde{\mathfrak{A}}$, and $\tilde{\mathfrak{A}}^{(1)}$, attenuating power of the medium; R', radius of cylinder; H, dimensionless height of the cylinder; Bu, Buger number; z, ρ, dimensionless coordinates of a point; $\Delta\theta$, dimensionless temperature jump on the curved surface of the cylinder.

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HEATING OF SOLID SURFACES BY AN ELECTRIC ARC

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We have developed a calorimetric probe for measuring unsteady heat fluxes with a resolving time $\Delta t < 10^{-3}$ sec. We have determined the flux to solid surfaces from an electric arc stabilized by a rotating cylinder.

An increase of the efficiency of high-temperature technological processes in many cases requires the use of intense sources for heating materials with a long operative life. Thanks to successes in producing jet arc plasmatrons, the problem of obtaining continuous heat flux densities $q < 5 \cdot 10^3$ W/cm² can be considered solved. Higher fluxes are achieved with a relatively low coefficient of utilization of the energy input [1-3]. In addition, the large dynamic head on the surface of a heated body limits the use of jet generators to solve problems of the heat treatment of materials when a surface film of molten material is present.

An arc plasma has a higher temperature and a lower flow velocity than a plasmatron jet. In view of this, it is of interest to investigate the creation of devices for heating the surfaces of bodies directly by an electric arc. The construction of one such plasmatron is described in [3]. In the present article we report the results of a study of the surface heating of bodies by using a similar device whose mode of operation is explained in Fig. 1a. The ends of the plasmatron electrodes were arranged in such a way that the plasma column was oriented parallel to the surface being heated 3. Its position in space is fixed by the rotating cylinder 1, mounted above the surface being treated at a distance d, less than the diameter of the current-conducting column. Because of the viscosity of the surrounding medium, a rotating gas stream is formed around the cylinder which clamps the plasma column simultaneously to the surface of the body and the cylinder.

The temperature of the gas layer between the cylinder and the plasma column is determined largely by the characteristics of the surrounding medium. Since the device operates in the open atmosphere, the temperature of the air layer will be relatively low, and thermal and electrical contacts between the rotating cylinder and plasma are negligible. It was shown experimentally that for currents in the range 30-80 A the arc is shunted onto a current-carrying cylinder 15 mm in diameter only at low rotational velocities $n \leqslant 5$ rps. The discharge is not shunted at high velocities even for quite long ($l > 100$ mm) cylinders. This permits a substantial simplification of the construction and an improvement of the operational characteristics of the device to position the arc by replacing the dielectric cylinder with a metal one. The surfaces of large articles are heated by displacing the plasmatron with a special mechanical device in a direction perpendicular to the axis of the arc.

The intensity of heating of samples was studied by a calorimetric measurement of the heat flux supplied to their surfaces. In most cases such measurements were performed with a probe in the form of a copper rod with a thermocouple pressed into it. The resolving time is determined by the distance from the collecting surface of the probe to the thermocouple. However, the indeterminacy of the position of the heat-sensitive layer in the body of the calorimeter, and the presence in it of appreciable voids, even with tight calking of the thermocouple with two wire outlets, prevented highly accurate measurements of intense unsteady heat fluxes. Additional calibration experiments have their own errors, and therefore do not permit a significant increase in the accuracy of the determination of the heat flux. A major flaw of the probe described is its slow response. As a consequence of the large size

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